

# ENGINEERING PERSPECTIVE FOR THE SEISMIC SITE RESPONSE OF ALLUVIAL VALLEYS

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## SUMMARY

The seismic site response of alluvial valleys with limited width is evaluated using three engineering models. The models are based on the one-dimensional, two-dimensional and the frame model approaches. The objective is to analyse the effects of the main parameters governing surface motions and provide engineering guidance for predicting them. The limitations on the use of the one-dimensional model in site response evaluation in valleys are pointed out. The frame model, which accounts for the limited width of valley, gives response results that are in good agreement with the two-dimensional model results. It is found that the effect of the two-dimensional amplification is significant over a distance from the valley edges beyond which the response may be adequately represented by one-dimensional analysis. The soil amplification varies depending on the soil type, site location relative to the valley and the dominant period and amplitude of input rock record. © 1997 by John Wiley & Sons, Ltd.

KEY WORDS: earthquake; site response; dynamic analysis; valley; soil conditions

## 1. INTRODUCTION

The characteristics of earthquake shaking at a given site during a particular event depends on a number of factors including: the source mechanism, the distance and geological characteristics of the rocks from source to site, wave interference and local soil conditions at the site. Analysis of the source mechanism and the effects of transmission path geology on earthquake waves is an important area of seismology. The effects of local site conditions on the characteristics of ground motions have been the focus of research by both seismologists and geotechnical engineers. The aim of these studies is to define the design earthquake which is of interest to the structural engineer.

Soil parameters and geologic conditions that may have significant effect on the amplification of ground motion at a site include: depth of soil layers above bedrock, variation of soil type and properties with depth, lateral irregularity and surface topography at the site. The response of soil deposits underlain by sloping rock boundaries to travelling wave base excitation was studied by Dezfulian and Seed.<sup>1</sup> The finite lateral extent of soil surface layers generates surface waves at the edge and two-dimensional resonance patterns in the lateral direction and increases the amplitude as well as the duration of ground motion.<sup>2,3</sup> This creates spatially varying ground motions at the surface of a sediment-filled valley which are of particular significance in the assessment of the seismic response of long structures such as dams, bridges or life-line systems.

Apparent differences in the main interests of seismological and geotechnical engineering studies have been reported by Finn.<sup>4</sup> For example, geotechnical engineers recognized the importance of non-linear effects at most soil sites during strong shaking, following the pioneering studies of ground response during the Niigata earthquake of 1964 by Seed and Idriss.<sup>5</sup> On the other hand, seismological studies tend to find a good correlation between weak and strong motions at a given site, namely, similar amplification for both, implying that non-linearities are not important as the first-order effect in most cases.<sup>2</sup> Reviews by Sanchez-Sesma,<sup>6</sup> Aki<sup>2</sup> and Faccioli<sup>7</sup> have shown that the effects of surface and buried topography on ground motions can be significant from both the seismological and engineering points of view. However, practical engineering

applications have mainly relied on the one-dimensional analysis to predict surface motions at any site, overlooking the effects of surface topography and the limited lateral extent of soil in sediment-filled valleys.

In this study, three engineering models are reviewed for the valley seismic response evaluation: the one-dimensional (1-D) model, the two-dimensional (2-D) finite-element model and the frame model developed by Rassem *et al.*<sup>8</sup> The frame model represents a simple practical engineering tool that takes into account the effect of the limited width of the valley. The frame model was applied to analyse the dynamic response of a suspension bridge to spatially varying excitation at its supports.<sup>9</sup> The 1-D approach is based on the assumption that the main response in soil is caused by upward propagation of shear waves from the underlying base rock.<sup>10</sup> Program SHAKE<sup>11</sup> is employed in this study to calculate the 1-D response. The finite-element approach is believed to provide the most accurate analytical solution to the dynamic response problem of the valley. A finite-element computer program FLUSH<sup>12</sup> is used for the 2-D response evaluation. The frame model was developed by modifying the 1-D model to consider the effect of the limited width of the valley thus providing a simple and realistic response prediction for symmetric valleys. The above three models incorporate non-linear soil behaviour in a linear iterative scheme.<sup>5</sup>

The objectives of the present investigation are to provide guidelines on the benefits and limitations of using the one-dimensional frame model approach to obtain the free-field motions in valleys with limited width and to identify the effects of the main parameters governing the spatial variations of surface seismic motions in alluvial valleys. The influence of variation in the valley dimensions on the seismic site response as well as on the validity of the models is analysed. The effects of soil type and input rock motion characteristics are investigated.

## 2. OVERVIEW OF THE FRAME MODEL OF VALLEY

The frame model was developed as a simple engineering tool to predict the non-linear seismic response of symmetrical alluvial valleys which are bounded by rigid rock and have limited width. The rigid rock boundary is assumed not to be energy transmitting. In the model, the soil continuum is assumed to consist of horizontal layers, each of which is homogeneous and isotropic. The control motion in rock is selected as an acceleration time-history representing a horizontal (or vertical) component of excitation. The new model was introduced to compensate for the deficiency of the regular one-dimensional approach without sacrificing its simplicity. The separation of the valley mode shapes into variations with depth and variations with the horizontal direction led to the development of the model.<sup>8</sup> To evaluate the horizontal valley response to a given horizontal rock motion, the frame model was obtained by replacing the second dimension of the valley by a set of horizontal axial links attached to the lumped masses of the one-dimensional model. The horizontal links introduce the effect of the limited valley width to the 1-D model by representing the lateral axial stiffness created by the finite lateral extent of soil in plane. In the analysis, the angle of incidence of the rock motion is not explicitly included. However, since engineering interest is in the horizontal and to a lesser extent, the vertical components of ground motion at the structure, the general direction of rock motion can be resolved into horizontal and vertical components before input into the frame model.

The valley shown in Figure 1 consists of  $N$  horizontal soil layers and may be idealized as the frame model illustrated in Figure 2. Due to symmetry, Figure 2 shows only one-half of the frame. The subscript  $m$  denotes

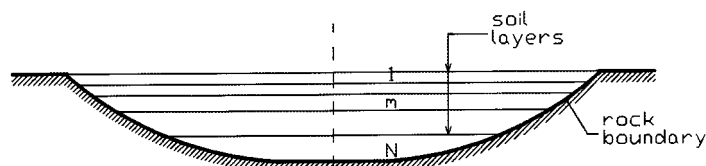


Figure 1. Symmetric valley

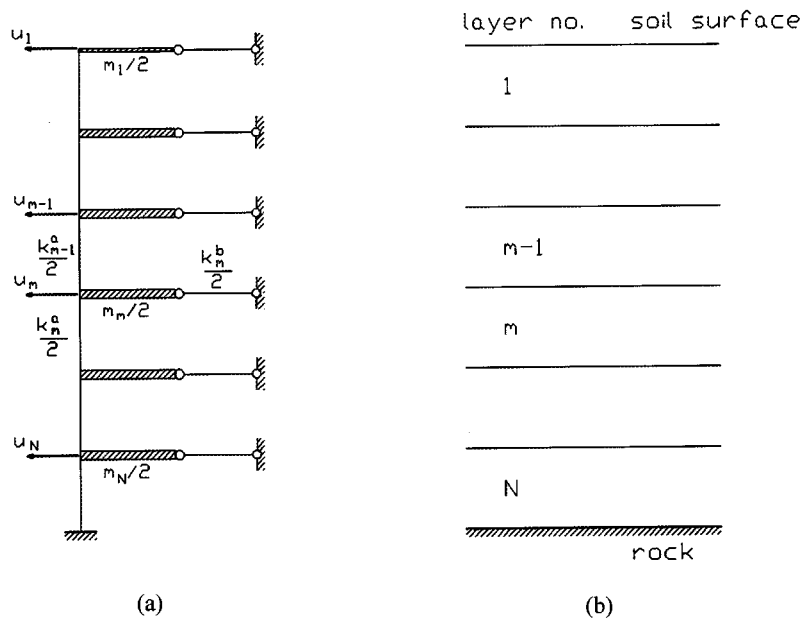


Figure 2. The frame model of the valley: (a) frame model; (b) layer profile

the layer number ( $m = 1, 2, \dots, N$ ). The soil-layer masses,  $m_m$ , are lumped at the top of each layer and connected to each other by vertical columns which represent the lateral shear stiffness properties of the soil,  $k_m^a$ . The masses are connected to the sloping rock boundaries by horizontal axial links representing the lateral axial stiffness,  $k_m^b$ , resulting from the limited horizontal extent of the soil. The soil masses can only move in the lateral direction, which means one horizontal degree of freedom for each mass,  $u_m$ .

SAP IV computer program<sup>13</sup> was modified to accommodate the new model and calculate the horizontal response by mode superposition in the time domain. The modal participation factors associated with the frame modes were modified to include the effect of the second dimension that was added to the frame model.<sup>8</sup> The computer code was also adjusted to permit different damping ratios for different modes, calculated from the contribution of the damping of each component of the system to the modal values.<sup>14,15</sup> The resulting motions from the frame model reflect the valley response at the centroidal axis. The response at other points in the valley can be obtained by utilizing the modal variations in the horizontal direction. The non-linear behaviour of soil was accounted for by using the equivalent linear method,<sup>5</sup> where a linear iterative procedure is followed to ensure that shear modulus and damping values are compatible with the effective strains in each small linear step.

### 3. INFLUENCE OF VALLEY DIMENSIONS

For simplicity, consider a symmetrical valley of trapezoidal cross-section with various dimensions, from wide and shallow to narrow and deep, along with different slopes of rock boundary. The effect of changes in the dimensions  $B$ ,  $D$  and  $L$  (Figure 3) of the valley on the horizontal surface seismic motions as well as on the validity of the engineering models (1-D, 2-D and frame) will be examined. The examples provided are for soil types (NC) and (DS), which are representative of a soft profile (normally to lightly consolidated clay) and a stiff profile (dense sand), respectively. The  $G_0$  profiles versus the depth  $z$  of the two soil types are reproduced from Elhamadi et al.<sup>16</sup> in Figure 4, where  $G_0$  represents the value of the shear modulus of soil at very low shear strain levels. The curves showing the relationship between  $G/G_0$  and shear strain,  $\gamma$ , in Figure 5, are

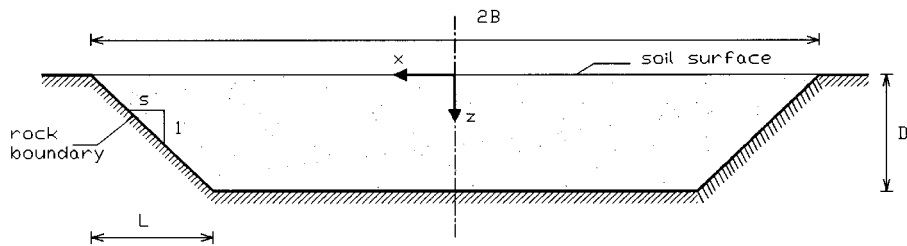
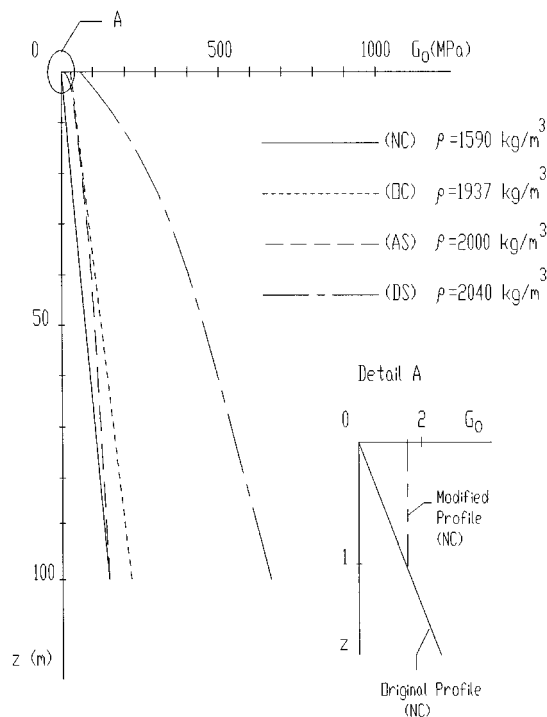


Figure 3. The valley geometry

Figure 4.  $G_0$  profiles of four soil categories

those recommended by Pappin *et al.*<sup>17</sup> The curves proposed by Seed and Idriss<sup>18</sup> and shown in Figure 6 are used to describe the damping ratio versus shear strain relationships for clay and sand profiles.

Two of the significant characteristics of earthquake motions are the frequency and amplitude of motion. These two parameters can be represented by means of a response spectrum. While response spectra do not provide information about the duration of the ground motion, they are commonly used by engineers for determining design earthquake motions. This study will thus focus on response results in the form of acceleration spectral ratios at different locations of the valley to reflect the soil amplification of rock motion. The ratio ( $a/v$ ) of peak acceleration,  $a$  (in g), to peak velocity,  $v$  (in m/sec), of an earthquake record is a quantitative measure of its frequency content. Three acceleration response spectra are created and shown in Figure 7. The high (H), intermediate (I) and very low (V)  $a/v$  ratios are representative of earthquake records

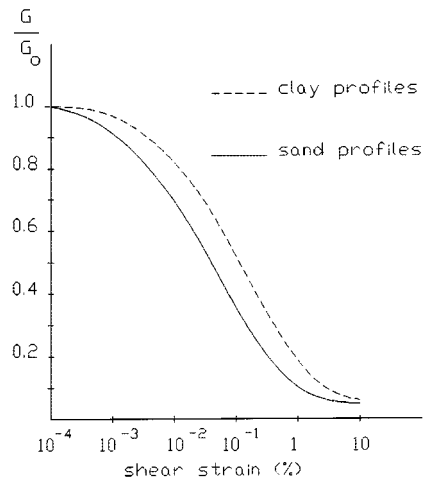
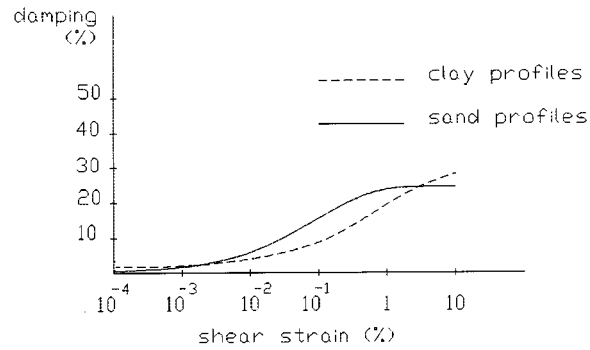
Figure 5. Design values of  $G/G_0$  for various soils

Figure 6. Damping ratios for clay and sand profiles

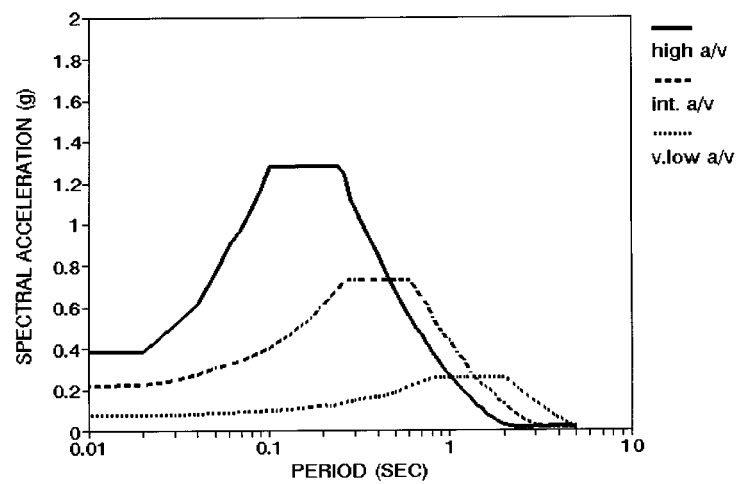
Figure 7. Acceleration spectra of input rock motions — damping ratio = 2 per cent scaled to peak velocity  $v = 0.2$  m/sec

Table I. Fundamental periods of the valley  
(normally consolidated clay (NC), very low  $a/v$ ,  $v = 0.2$  m/sec)

$D$ (m)	$L$ (m)	$T$ (sec)				
		$B = 100$ m	$B = 300$ m	$B = 500$ m	$B = 1000$ m	$B = \infty$
20	100	1.18	1.35	1.35	1.35	1.35
	0	1.73	1.88	1.89	1.90	1.90
40	100	1.45	1.87	1.89	1.90	1.90
	200	N/A	1.86	1.89	1.90	1.90
100	100	1.75	2.57	2.70	2.73	2.73

D valley depth

L horizontal length of the sloping rock boundary of the valley

B half breadth of the valley

with high, intermediate and very low dominant frequencies, respectively. Three artificial acceleration time-histories are generated to match the proposed three target response spectra shown in Figure 7. The generated acceleration time-histories are used as horizontal input excitations at the rock boundary, thus covering a wide variety of frequency content ranges of actual earthquake records. The smooth shapes of the spectra allow more consistent investigation of the valley effect on the amplification of rock motions.

The soil surface response is calculated using the frame model. It is then compared for some cases with the response from the 2-D model (program FLUSH) and 1-D model (program SHAKE). The period at which the largest peak of amplification occurs at a site is termed the local-amplification period of the site. The variation in the fundamental period of the valley is given in Table I for soil profile (NC). The case of  $B = \infty$  corresponds to the resulting fundamental periods when the limited valley breadth is ignored in the 1-D analysis. It is evident that the fundamental periods are more sensitive to the change in the depth  $D$  than to the variation in  $B$  or  $L$ . This is found to be typical for other soil types.

The site-amplification curves for various values of valley depth are plotted in Figure 8. Increasing the depth  $D$  leads to longer local-amplification periods of sites because of the decrease in the overall stiffness. In general, the variation in depth leads to significant shifts in the fundamental period of the valley and some changes in the magnitude of amplification peak at the valley centre. The top layers of dense sand (DS) sites with small depths are subjected to low shear strains. Low shear strains are associated with low damping values and high amplification. Hence, smaller depths result in larger top-layer amplification in the short-period range as shown in Figure 8. The increase in the depth  $D$  leads to a larger sloping rock area when the valley width is kept unchanged. Hence, larger depths result in higher amplification peaks at or near the middle of the valley as shown in Figure 8(d), due to more focusing of the incoming waves from the sloping rock boundary toward the valley centre. These results agree with Frankel's<sup>19</sup> suggestion that multiple reflections above the sloping interface produce trapped surface waves that propagate into the centre of the basin. This also explains the amplification peaks being higher near the centre than at sites near the edge of the valley, as shown in Figure 9. The amplification peaks increase from one surface point to another until point d is reached, from the left edge, where the highest effect of the two-dimensional amplification is obtained. The position of this site depends on the  $L/B$  ratio and the depth  $D$ . Beyond this site, as the centre of the valley is approached, amplification peaks may slightly decrease for a distance, as the lateral amplification effect diminishes, after which they remain unaltered and the one-dimensional approach may apply. In general, effect of the two-dimensional amplification becomes more pronounced as the valley approaches a triangular shape by decreasing  $B$  or increasing  $D$  or  $L$ . It is further observed from Figure 9 that the periods associated with the local-amplification peaks differ from one site to another, depending on the relative location in the valley. The local-amplification period at the middle surface point of the valley is the longest and

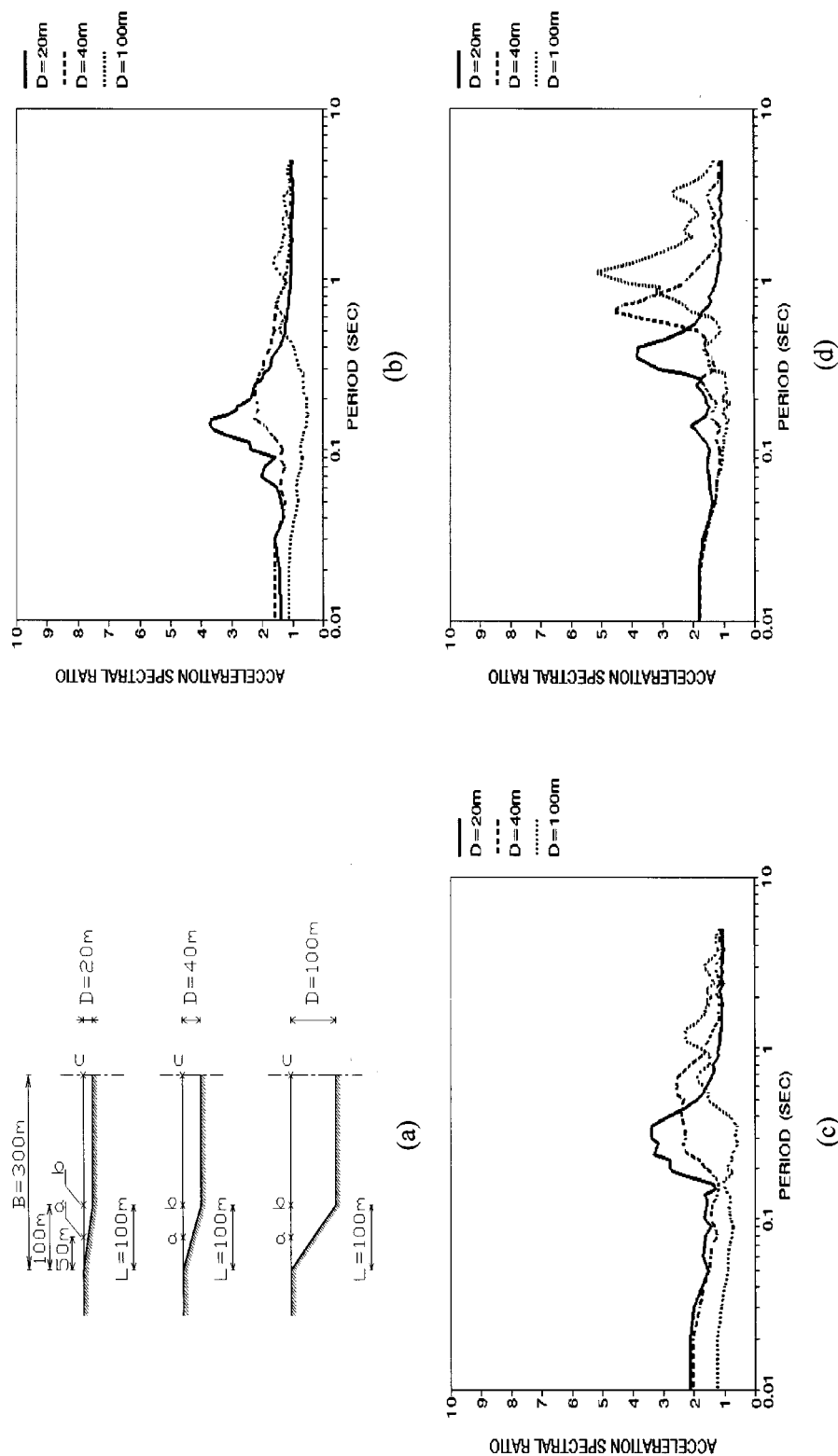


Figure 8. The effect of change in depth on the site amplification. Soil profile: DS,  $a/v = 1$ ,  $v = 0.2\text{ m/sec}$ ; (a) valley dimensions; (b) site a; (c) site b; (d) site c

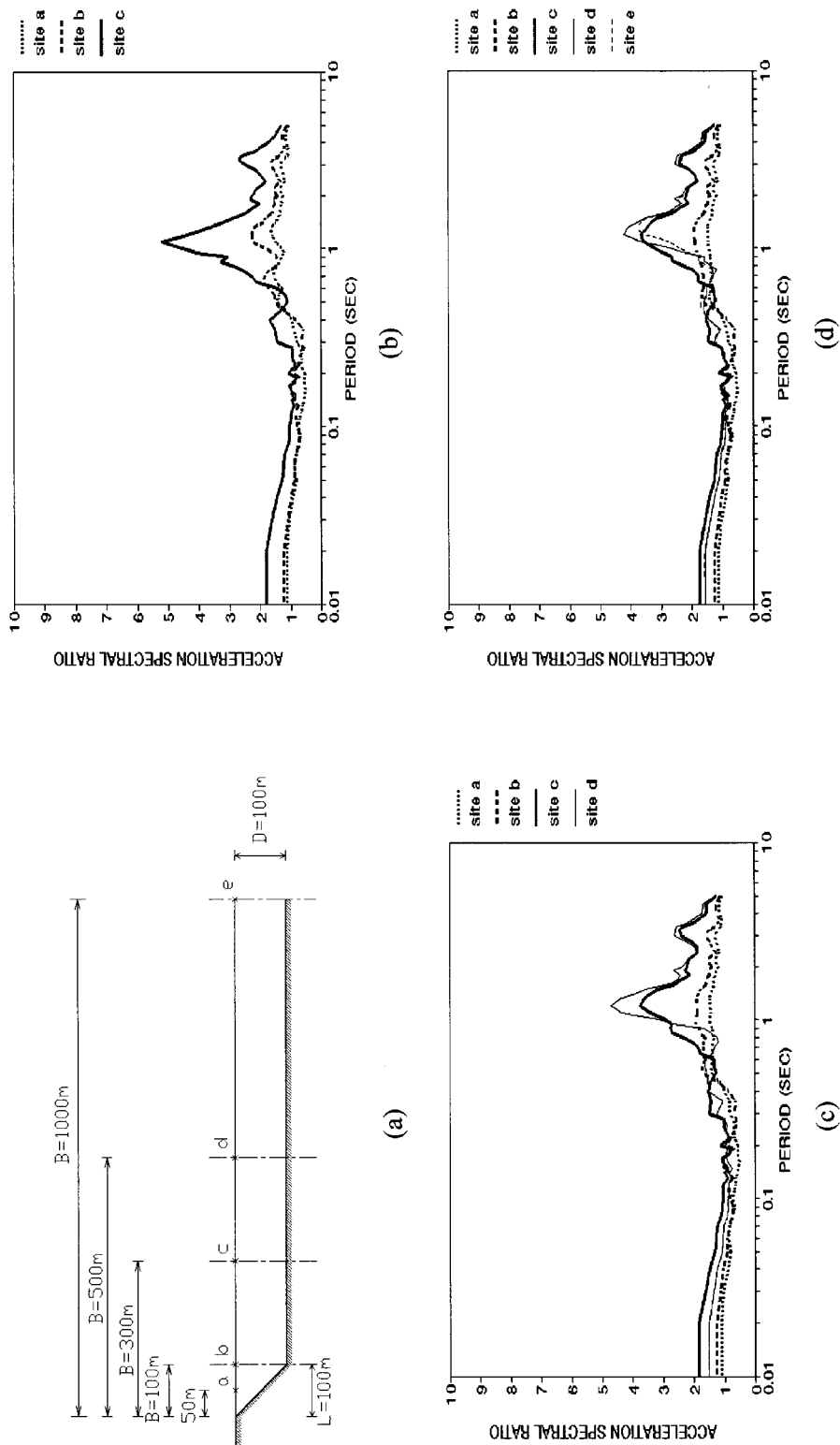


Figure 9. Variation of site amplification at different locations in the valley. Soil profile: DS,  $D = 100\text{ m}$ ,  $a/v = I$ ,  $v = 0.2\text{ m/sec}$ ; (a) valley dimensions; (b)  $B = 300\text{ m}$ ; (c)  $B = 500\text{ m}$ ; (d)  $B = 1000\text{ m}$



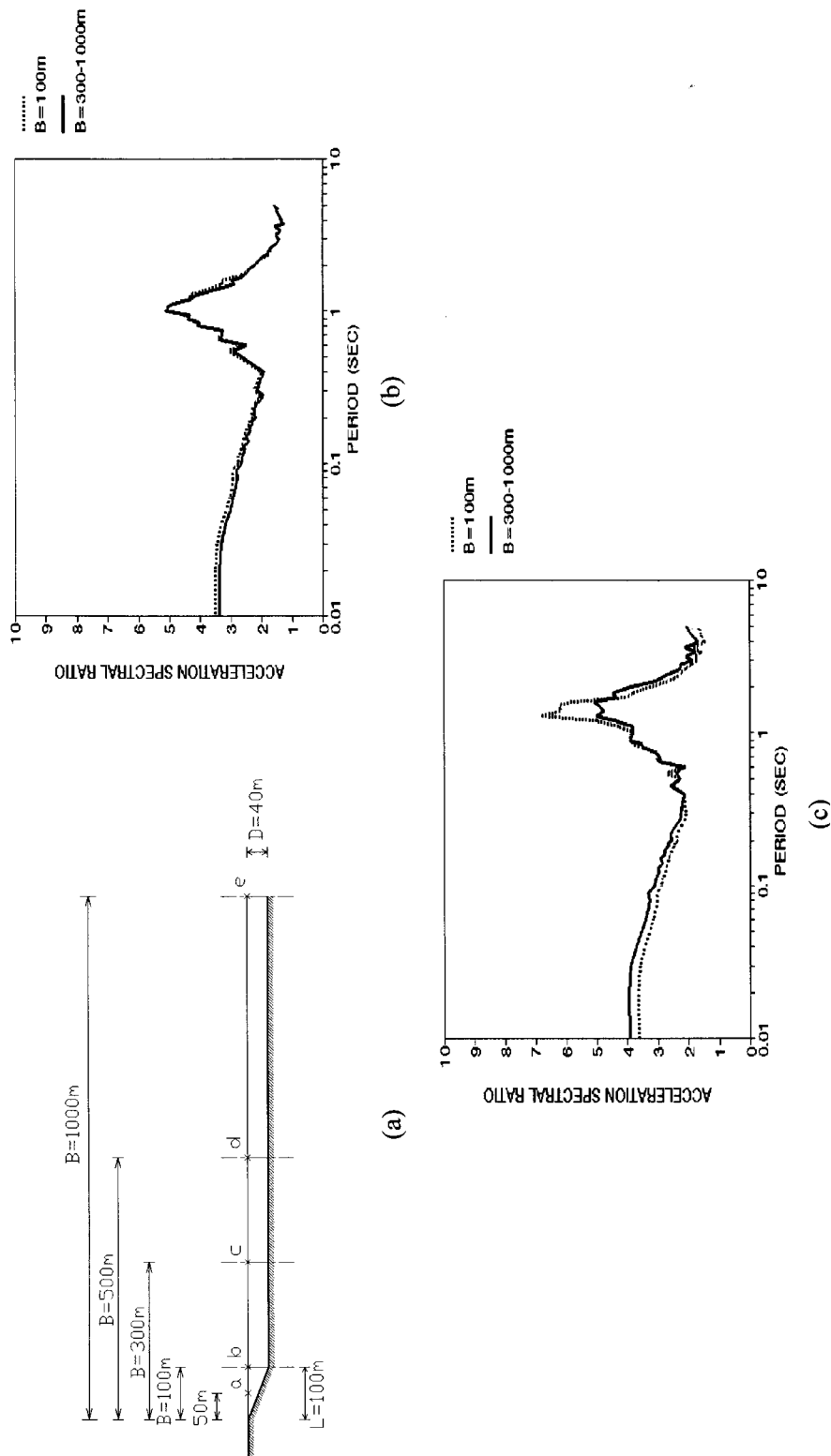


Figure 10. Effect of the change in breadth on the site amplification. Soil profile: NC,  $a/v = V$ ,  $v = 0.2\text{ m/sec}$ : (a) valley dimensions; (b) site a; (c) site b

thus coincides with the fundamental period of the valley. As the selected site approaches the edge of the valley, the local-amplification period becomes shorter due to the stiffening effect from the sloping rock boundary.

Increasing the valley width  $B$  from 300 to 1000 m does not affect the amplification at the edge surface points  $a$  and  $b$ , as shown in Figure 10. This means that the amplification at the edge sites is mainly affected by the incoming waves from the surrounding area (the sloping rock boundary). In other words, for relatively wide valleys ( $B \geq 300$  m) the edge-site amplification is a function of the depth and the slope underneath and is independent of the breadth  $B$ .

It is considered that the 2-D finite-element model of the valley provides a reliable analytical approximation to the surface response. To judge the validity of other models, the resulting acceleration spectral ratios from the 1-D and the frame models are compared with those from the 2-D finite element model. The 1-D model provides poor approximation to the response at surface points near the edge of the valley as illustrated in Figure 11. These edge sites are the spots mostly affected by the sloping rock boundary. For narrow and deep valleys ( $B/D < 10$ ), the response approximation provided by the 1-D model is not good even at surface points right in the middle of the valley, as evident from Figure 12. The 1-D analysis underestimates the response predictions in the middle of the deep valley, particularly in the neighbourhood of the fundamental period. This is because the two-dimensional effect is more visible in deep valleys and tends to increase the amplification near the centre of the valley. The only sites where the 1-D analysis succeeds in approximating the response of the 2-D model are those located near or at the centre of wide and shallow valleys ( $B/D \geq 10$ ). On the other hand, examination of Figures 11 and 12 indicates that the frame model succeeds in providing reliable response results when compared with the response from the 2-D model. This proved to be true for the symmetrical valleys under consideration, regardless of the surface point location. It is to be noted that considerable saving in computing effort can be achieved by using the frame model as compared to the finite element model. This is mainly because the second dimension of the valley is represented by a large mesh in the case when finite element is used, while it is only represented by links in the frame model. The simple frame model proved advantageous when the site response is coupled with complex structural analysis<sup>9</sup>.

#### 4. INFLUENCE OF SOIL TYPE AND ROCK MOTION VARIATIONS

Four soil categories, with various stiffness variations along soil depth, are used in the analysis. The soil profiles in an increasing order of stiffness are: normally to lightly consolidated clay (NC), alluvial sand and silt (AS), heavily overconsolidated clay (OC) and dense sand (DS). The  $G_0$  profiles of the four soil types are shown in Figure 4. Influence of input rock motion characteristics on the seismic response includes the effect of frequency content and amplitude of motion. The three generated records, with high ( $H$ ), intermediate ( $I$ ) and very low ( $V$ )  $a/v$  ratios will be utilized as horizontal uniform motions in the rock to examine the effect of changing the frequency content on the seismic response. The peak amplitude of motion may be used as an indicator of the motion intensity level for the same frequency content and duration of motion.

A trapezoidal valley having dimensions:  $B = 300$  m,  $D = 40$  m and  $L = 100$  m is selected. Sites  $a$ ,  $b$  and  $c$  are located at distances 50, 100 and 300 m from the left edge of the valley. The horizontal surface motions at these sites are obtained using the frame model. The seismic response is presented in the form of surface spectral accelerations and spectral ratios, calculated at a critical damping ratio of 2 per cent. The spectral acceleration is a measure of the intensity of surface motion while the spectral ratio defines the soil amplification of rock motion. The period at which the largest peak of spectral acceleration occurs is the dominant period of the resulting surface motion. The dominant period at a site depends on both the dominant period of the input rock motion and the local-amplification period at that site.

The intensity of surface motion for different soil profiles is illustrated in Figure 13 when the valley is subjected to rock motion with high  $a/v$ . It is observed that the proximity of the local-amplification period at a site to the dominant period of the input rock record increases the resulting surface motion intensity. This accounts for the higher peaks of spectral accelerations in the case of stiffer soils (e.g. DS and OC) when  $a/v$  of

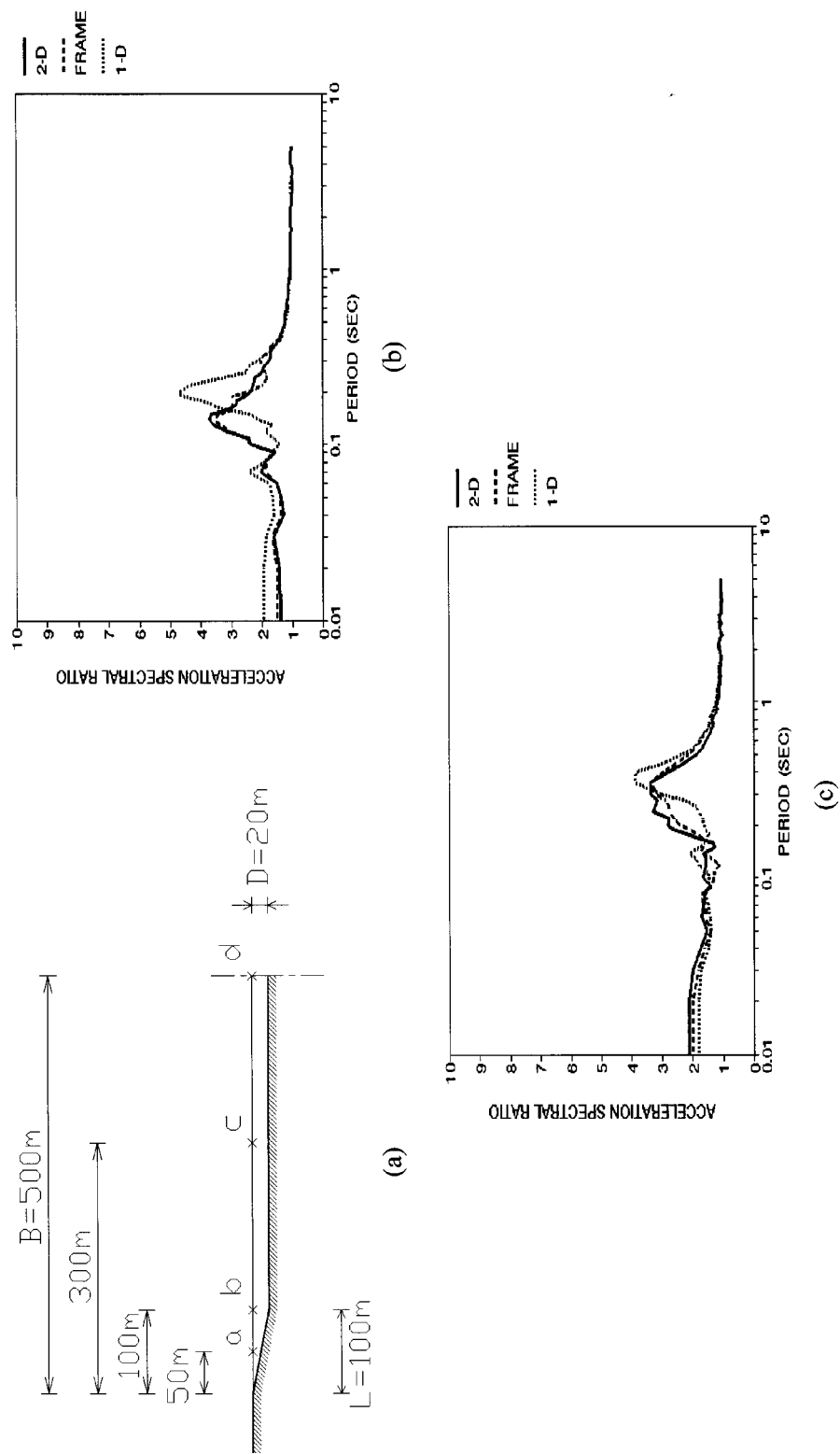


Figure 11. Validity of the engineering models at edge sites ( $B/D = 25$ ). Soil profile: DS,  $a/v = 1$ ,  $v = 0.2\text{ m/sec}$ : (a) valley dimensions; (b) site a; (c) site b

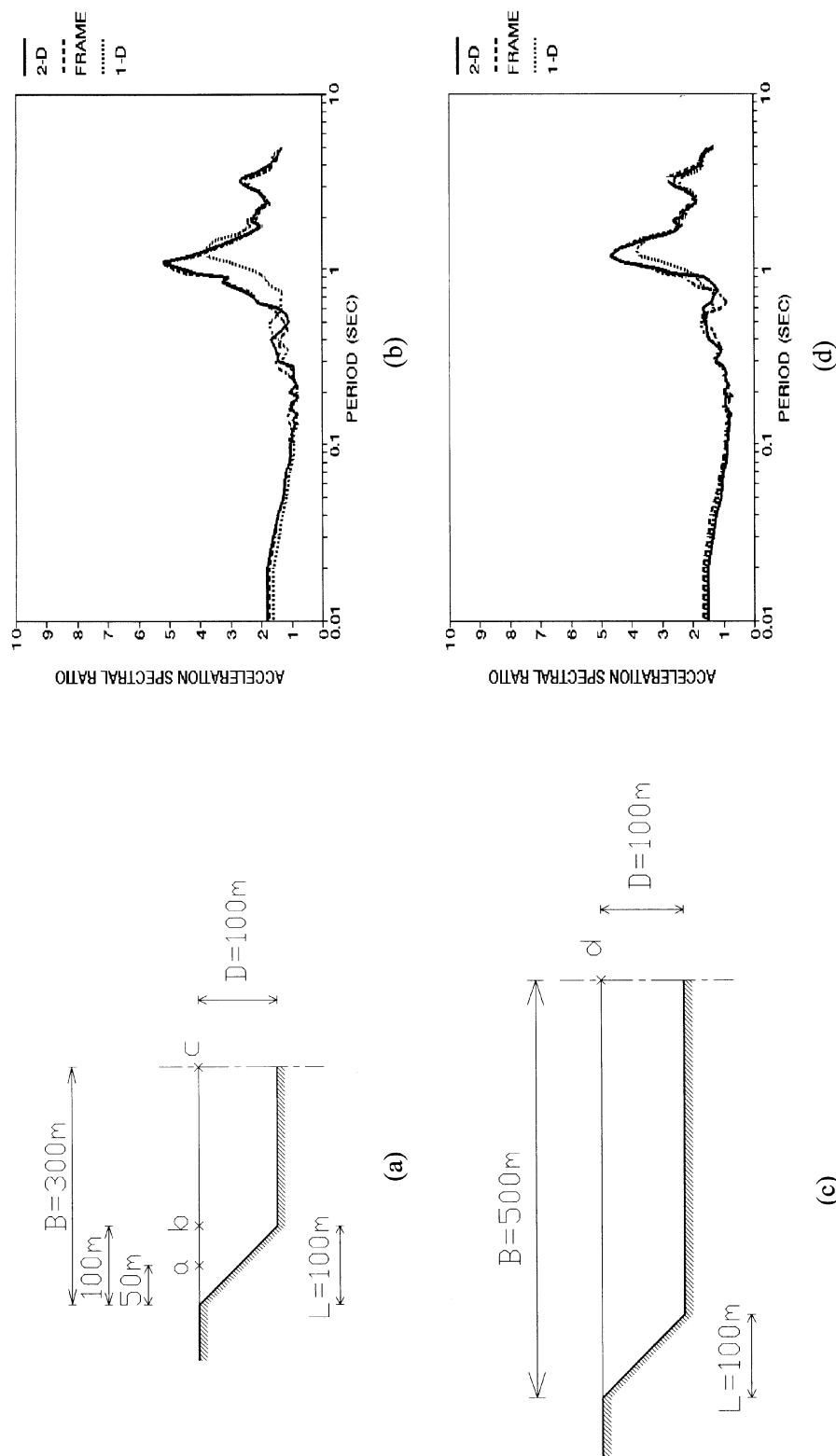


Figure 12. Validity of the engineering models at centre sites ( $B/D < 10$ ). Soil profile: DS,  $a/b = I$ ,  $v = 0.2\text{ m/sec}$ ; (a) valley dimensions; (b) site  $c$ ,  $B = 300\text{ m}$ ,  $D = 100\text{ m}$ ; (c) valley dimensions; (d) site  $d$ ,  $B = 500\text{ m}$ ,  $D = 100\text{ m}$

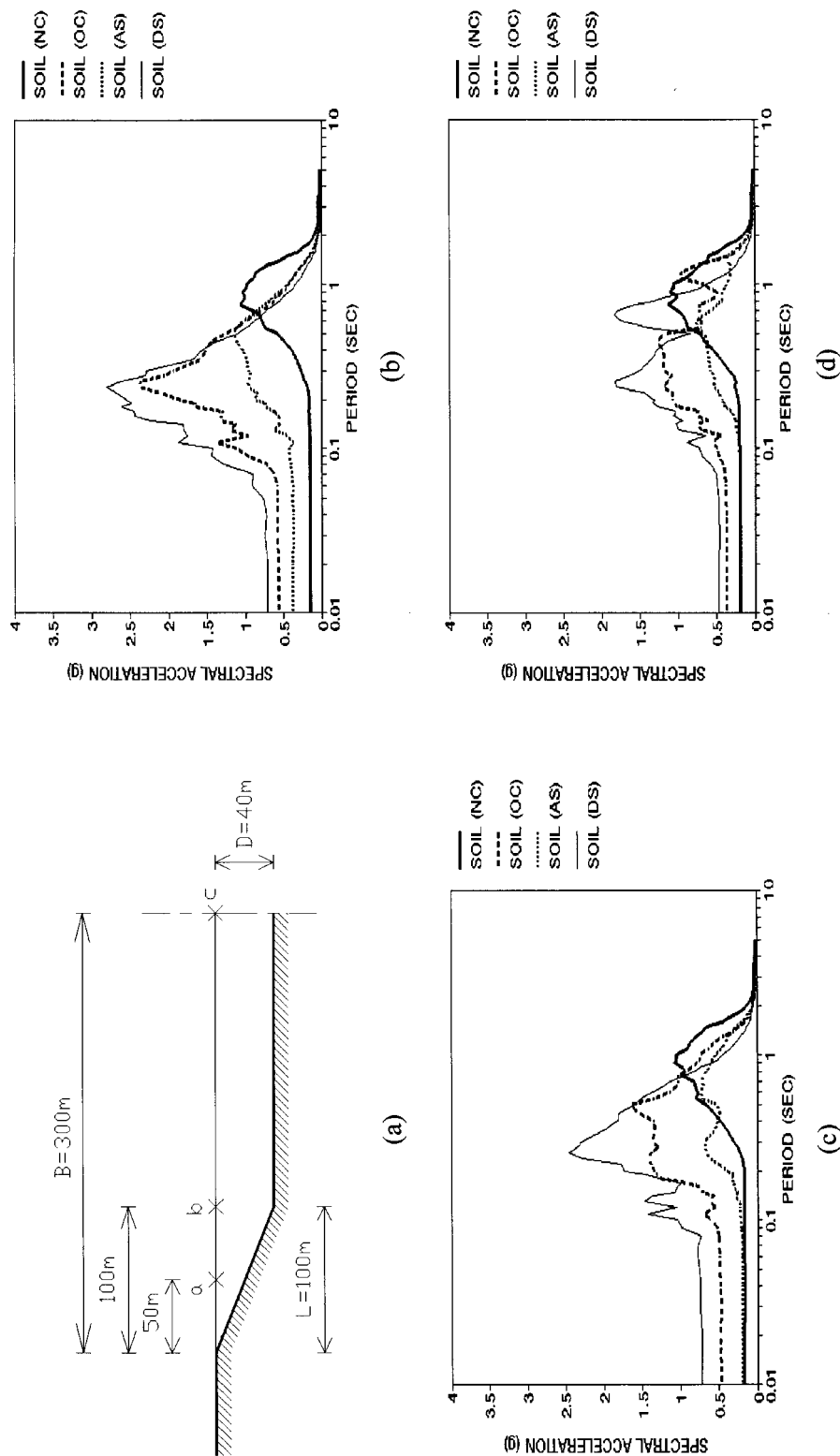


Figure 13. Effect of soil type on the spectral accelerations. High  $a/v$ ,  $v = 0.2$  m/sec: (a) valley dimensions; (b) site a; (c) site b; (d) site c

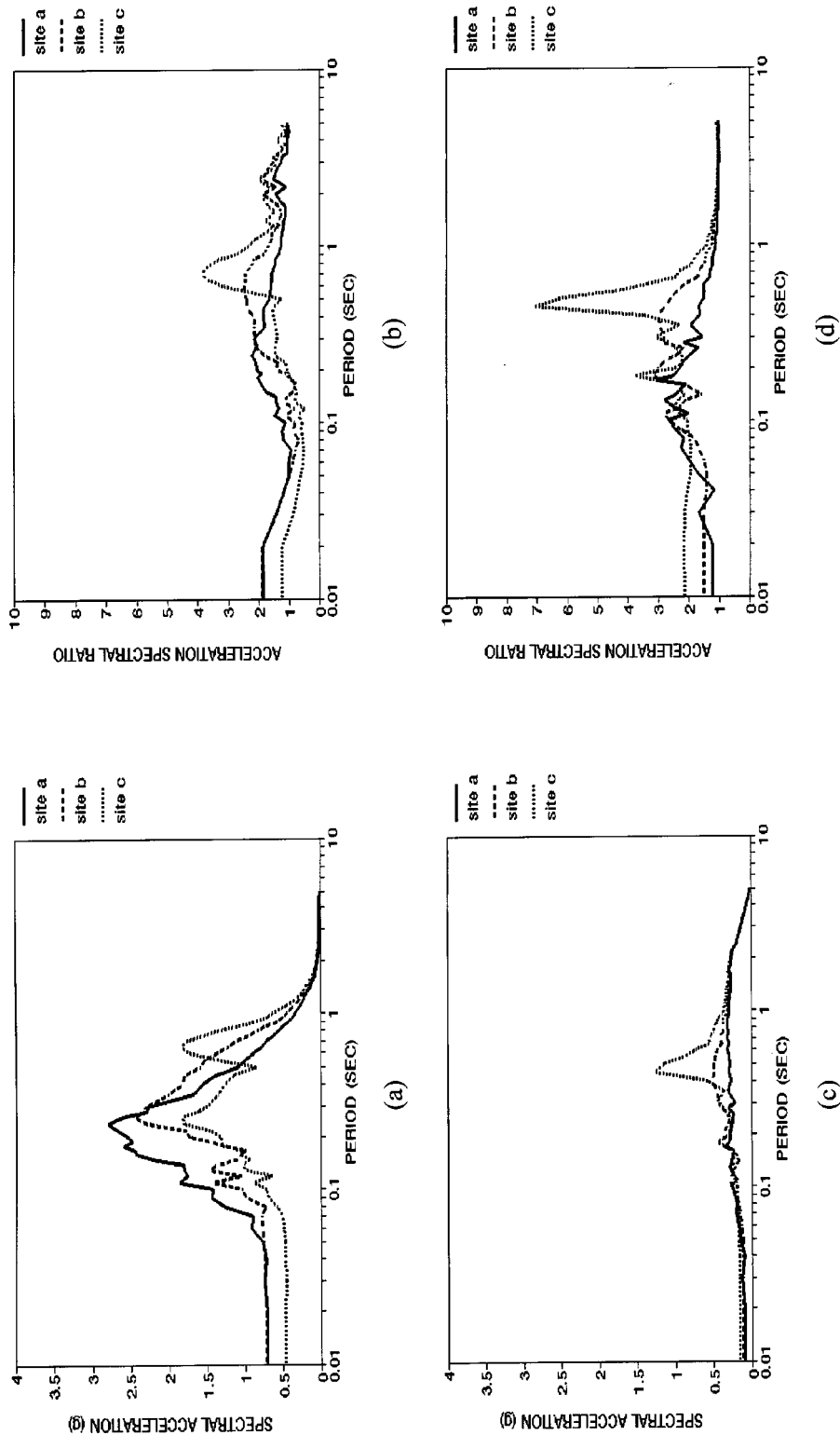


Figure 14. Effect of site location on the spectral response. Soil profile (DS):  $v = 0.2$  m/sec (a) spectral acceleration,  $a/v = H$ ; (b) spectral ratio,  $a/v = H$ ; (c) spectral acceleration,  $a/v = V$ ; (d) spectral ratio,  $a/v = V$

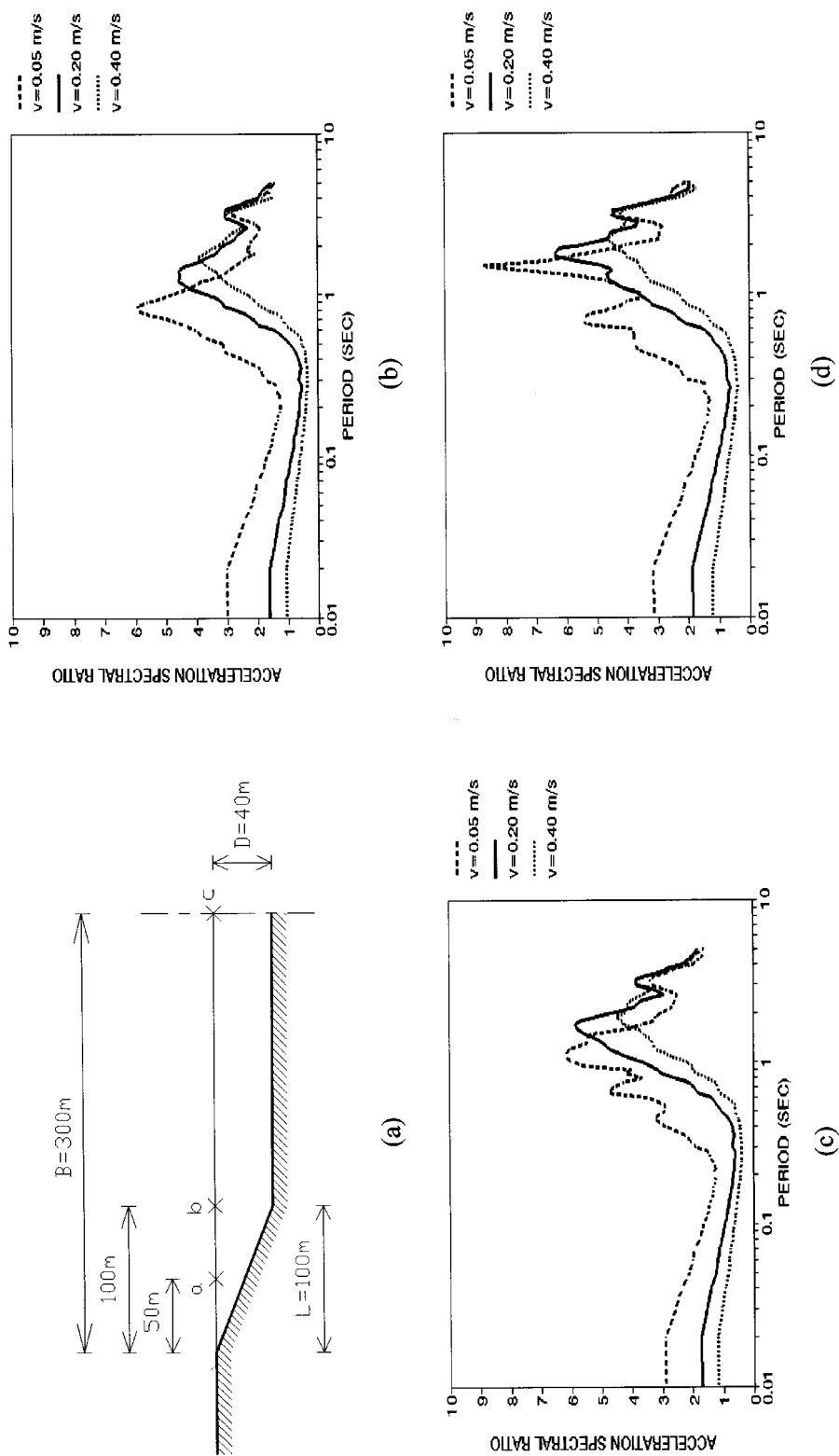


Figure 15. Effect of rock motion intensity on the acceleration spectral ratios. Soil profile (NC),  $a/b = 1$ : (a) valley dimensions; (b) site a; (c) site b; (d) site c

the input record is high, as shown in Figure 13. On the other hand, the softer profiles (e.g. NC) develop higher peaks of spectral accelerations for the low  $a/v$  input record.

Figures 14(a) and 14(c) show two contrasting patterns of behaviour for (DS) sites under rock motions with two different  $a/v$  ratios. In Figure 14(a) the magnitudes of spectral acceleration peaks increase from site c in the middle of the valley to site a near the edge, under high  $a/v$  ratio. In Figure 14(c) the magnitudes of spectral acceleration peaks decrease as the selected site approaches the edge of the valley, under very low  $a/v$  ratio. The reason for the contrasting behaviour is that the stiffening effect from the sloping rock boundary increases as the edge of the valley is approached. This causes the local-amplification periods of edge sites to become closer to the dominant period of the high  $a/v$  input record which leads to higher intensity of surface motion at site a than that at site c in Figure 14(a). In the case of very low  $a/v$  input record, the local-amplification period at site c is closer to the dominant period of the input record, which causes higher intensity of motion at site c than that at site a, as shown in Figure 14(c). However, the magnitudes of amplification peaks in both cases of high and low  $a/v$  ratios become higher and the periods at which the peaks occur get longer as the middle of the valley is approached, as shown in Figures 14(b) and 14(d). The reason for this behaviour was discussed before under the influence of valley dimensions.

The effect of the peak amplitude of rock motion on the soil amplification values is studied by scaling the input record to three levels of peak velocity:  $v = 0.05, 0.20$  and  $0.40$  m/sec. As the peak velocity  $v$  increases, the intensity of motion in soil becomes higher for the same duration and frequency content of input record. This develops higher shear strains in the soil and leads to higher surface spectral accelerations. The higher shear strains are associated with higher damping values and more softening in the soil. Consequently, increasing the intensity of motion leads to lower amplification due to the higher damping. It also causes longer local-amplification periods at the sites due to the smaller stiffness. This is illustrated in Figure 15 for soil profile (NC).

## 5. CONCLUSIONS

The site response of symmetric alluvial valleys has been investigated to provide practical and useful guidelines to engineers for determining design earthquake motions. The main parameters governing the spatial variations of surface motions in the valley were identified. These parameters include the valley dimensions, site location, soil type and input rock motion which were found to have significant effects on soil amplification and intensity of surface motion. The limitations on the use of the 1-D approach to obtain the free-field motions in valleys with limited width are demonstrated. The 1-D model can only be used in shallow and wide valleys with  $B/D \geq 10$  and  $B/L \geq 2$ , to obtain the surface response at sites located at least 10 times the depth  $D$ , away from the valley edge. These sites are beyond the effect of the two-dimensional amplification resulting from the sloping rock boundary. The frame model, which accounts for the effect of the limited width of the valley was used in the evaluation of the non-linear seismic response of symmetrical valleys. Response from the frame model is verified against the 2-D finite element solution and is shown to be in good agreement at all locations in the valley.

The local-amplification periods at different surface locations in the valley are found to be more susceptible to the change in the underlying depth of soil than to the change in other dimensions of the valley. The focusing of the incoming waves from the two sloping edges toward the valley centre results in increased two-dimensional amplification as the middle of the valley is approached from the edge up to a location beyond which the two-dimensional effect diminishes and the one-dimensional approach becomes applicable. Soil amplification at edge sites in the valley is independent of the breadth  $B$ , since edge sites are mainly affected by the incoming waves from the surrounding area. The edge sites of valleys with stiff soil are more vulnerable than the centre sites to rock excitations with high  $a/v$  ratio because the local-amplification periods at edge sites are closer to the dominant period of the high  $a/v$  rock records. This leads to higher intensity of surface motions near the edge.



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